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Field Wave Gaging Program, Wave Data Analysis Standard

by Marshal D. Earle, Neptune Sciences, Inc.

David McGehee, Michael Tubman, WES

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by Marshal D. Earle

Neptune Sciences, Inc.
11341 Sunset Hills Road
Reston, VA 22090

David McGehee, Michael Tubman

U.S. Army Corps of Engineers
Waterways Experiment Station
3909 Halls Ferry Road
Vicksburg, MS 39180-6199

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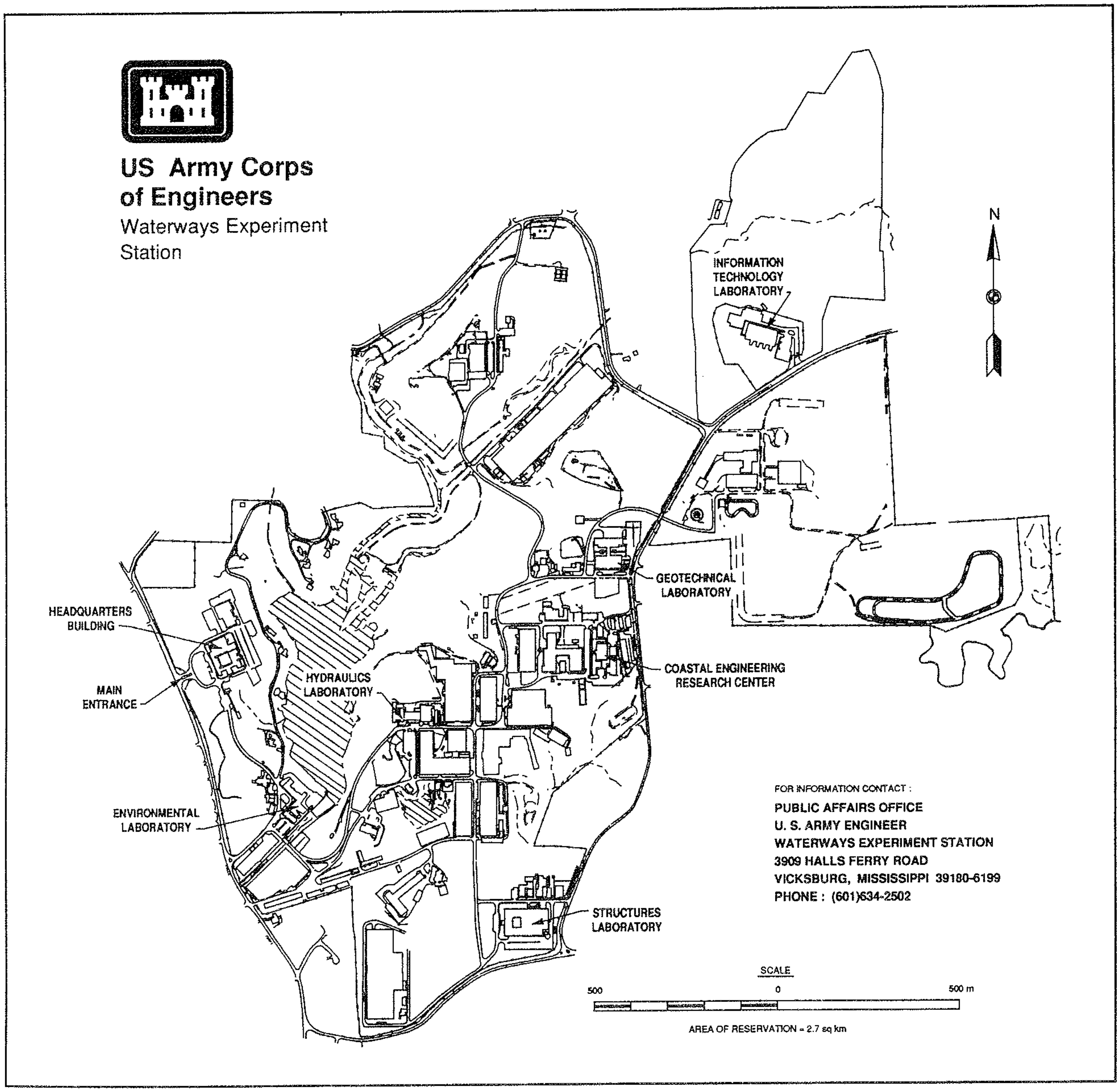

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Preface

This report is published by the U.S. Army Engineer Waterways Experiment Station (WES) Coastal Engineering Research Center (CERC). The work was funded by the Field Wave Gaging Program (FWGP), a work unit of the U.S. Army Corps of Engineers Coastal Field Data Collection Program (CFDCP). At Headquarters, U.S. Army Corps of Engineers, Technical Monitors for the CFDCP are Messrs. Charles Chesnutt, John H. Lockhart, and Barry W. Holliday. The Program Manager of the CFDCP is Ms. Carolyn Holmes, CERC; the Principal Investigator of the FWGP is Mr. David McGehee, CERC.

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1 Introduction

The U.S. Army Corps of Engineers uses wave data in planning, designing, and operating coastal projects. However, there is seldom enough time between the inception of a project and the need to use the data to collect sufficiently long records to establish wave conditions for engineering purposes. To address this problem, the Corps established the Field Wave Gaging Program (FWGP) to provide data to meet anticipated requirements. The FWGP is managed by the U.S. Army Engineer Waterways Experiment Station Coastal Engineering Research Center (CERC).

The FWGP obtains data principally from four gaging networks. The Coastal Data Information Program (CDIP) began in 1975 with a single station operated by the Scripps Institution of Oceanography (SIO), located literally in its back yard on the Scripps pier. The CDIP, under the joint sponsorship of the FWGP and the California Department of Boating and Waterways, currently operates 23 stations on the west coast and Hawaii. The Florida Coastal Data Network (FCDN) operated by the University of Florida (UF) also began in the mid 70's and grew to a peak of 10 stations around the Florida coastline. It is currently managed by the FWGP and the Florida Department of Environmental Protection. The largest network is the system of observation platforms operated by the National Data Buoy Center (NDBC). This network is primarily funded by the National Weather Service to support its forecasting mission. The FWGP supports, fully or partially, 20 of the NDBC network of 45 stations. CERC also operates an in-house network of 32 gages called the Network for Engineering Monitoring of the Oceans (NEMO). NEMO is operated by CERC's Prototype Measurement and Analysis Branch (PMAB) with support from FWGP and other Corps programs.

The majority of the wave data obtained by the FWGP is provided by the first three networks, which will be referred to as the FWGP networks. Their typical wave gage measures the sea surface through bottom-mounted pressure sensors either singly, combined into multi-sensor slope arrays, or in conjunction with a current meter measuring two orthogonal components of horizontal or orbital velocity (a PUV gage). Because of the attenuation of wave-induced pressure fluctuations with increasing depth and frequency, a pressure response correction is an integral part of the analysis procedure. The analysis procedures used to produce wave data from a pressure time series are similar to those used to analyze data from single or arrayed wave staffs, and data from surface-following buoys.

This version of the Field Wave Gaging Program Wave Data Analysis Standard focuses on procedures for analyzing directional wave data from pressure slope arrays and PUV gages. Though the FWGP utilizes data from NDBC buoys (as well as from buoys operated by the other networks,) the analysis procedures for buoy data are sufficiently unique, due to aspects of hull response and telemetry constraints, to be excluded from this document. Nondirectional wave data analysis is inherently included as a simplification (a subset) of directional wave data analysis. Likewise, analysis of data from wave staffs is a subset of these procedures, obtained by eliminating the pressure response correction. The data collection and analysis procedures described are only applicable to wind-generated surface gravity waves of engineering significance. This document covers analysis procedures for measured time series (wave records) that are assumed to contain no significant number of errors or gaps.

2 Purpose

To effectively use wave data statistics in solving engineering problems, it is desirable that they represent wave conditions in a manner consistent with the theories and techniques applied to the problems. Data in the FWGP database are from different sources, locations, and times. Variations in the methods used to make the measurements, analyze the data, and report the results affect the consistency of the data and its utility for application to engineering solutions. Applications requiring integration of data obtained from more than one network have been hampered by these differences.

To advance its mission of collecting and disseminating wave data for the U.S. coastline, the FWGP will issue a series of standards on wave data collection, analysis, and climate statistics. The purpose of this document is to provide specifications on the analysis techniques that must be applied to data obtained for the FWGP to ensure quality and uniformity of the final products. Development of a standardized database, in turn, will promote the accessibility and utility of the wave data. The standard has evolved through a consensual approach involving the principals of the four noted organizations, which are responsible for the vast majority of wave measurements for this country.

3 Present FWGP Procedures

Documentation Approach

Development of the FWGP wave data analysis standard began with documentation of present FWGP wave data analysis procedures. A working paper containing detailed mathematical descriptions of generally accepted wave data analysis procedures was prepared and used as a framework and reference at a workshop attended by representatives from each FWGP network in February 1993 (Tubman, Earle, and McGehee, in preparation). Information obtained from workshop participants and during subsequent discussions enabled documentation of present procedures. Results are summarized in this section. Appropriate mathematical background and details are provided to facilitate implementation of the wave data analysis standard by present as well as future FWGP networks. The procedures are compared and evaluated in Chapter 4, and a standard procedure is prescribed in Chapter 5. Definitions are provided in Appendix A.

Data Collection

This document covers wave data analysis, rather than wave data collection or instrumentation. Aspects of wave data collection are not described except as they directly relate to data analysis. For example, wave gage design and calibration are not covered. This document assumes that measured time series have been appropriately corrected for instrumentation-caused effects before data analysis.

Table 1 summarizes the most important present data collection parameters that pertain to subsequent data analysis. All three networks use PUV gages. The FCDN presently uses them exclusively while the CDIP and NEMO also use multiple pressure sensor slope arrays. There are minor differences in record lengths and storm-mode thresholds which are a result of regional differences in wave conditions. The CDIP uses longer record lengths and variable storm mode thresholds because long-period swell may occur off the west coast, and waves off the west coast are often larger than waves off the east

Table 1
Present Data Collection Parameters

Network	FCDN	NEMO	CDIP
Data points per record	1,024	1,024	Variable 1,024 - 8,192 2,048 typical
Sampling rate	1 *Hz*	1 *Hz*	Variable 0.125 - 2 *Hz* 1 *Hz* typical
Record interval normal mode	1 hr, collected 6 hr, retained	1 hr, collected 4 hr, retained	3 hr
Record interval storm mode	1 hr	1 hr	Continuous data for time period wanted
Storm-mode threshold	Variable $H_{mo} > 2$ m typical	$H_{mo} > 1$ m, Great Lakes $H_{mo} > 1.5$ m, U.S. east coast	Variable
Record start time	On the hour	On the hour	Variable, dependent on time to poll all stations
Reported record time	Record start time, local standard time	Record start time, Universal Coordinated Time	Record end time, Pacific Standard Time

coast and Gulf of Mexico. There are differences in intervals between data records in both normal and storm data collection modes and in the specific times that data records are collected. These differences do not affect analysis of individual data records. They should have negligible effects on climatological statistics. Extreme wave conditions can be identified using data from all three networks since record intervals are short during storm modes.

Data Analysis

Overview

The steps in wave data analysis are: (a) initial data quality assurance (DQA) tests to reject poor quality data, (b) data segmenting to reduce statistical uncertainties of spectra, (c) mean removal, (d) trend removal, (e) use of windows to reduce spectral leakage, (f) corrections for window use, (g) fast Fourier transforms (FFT's), (h) cross-spectral analysis, including segment averaging, (i) directional spectra calculations, (j) transformations of wave directions to output coordinate conventions, and (k) final determination of directional spectra and products derived from these spectra (e.g. nondirectional spectra and wave parameters such as significant wave height). Performing all steps after initial DQA in the same order is not necessary to obtain the same results given the same input data.

Kinsman (1965) provides useful philosophical and intuitive descriptions of wave data analysis concepts. Earle and Bishop (1984) describe wave analysis procedures involving statistics in an introductory manner. Several papers by Longuet-Higgins (e.g. 1952, 1957, 1980) are among the most useful papers which describe statistical aspects of ocean waves. Longuet-Higgins, Cartwright, and Smith (1963) provide a classic description of directional wave data analysis that is used for directional wave measurements by buoys, slope arrays, and PUV gages. Donelan and Pierson (1983) provide statistical results that are particularly useful for significant wave height. Dean and Dalrymple (1984) discuss theoretical and practical aspects of waves without emphasis on statistics.

Table 2 summarizes the data analysis methods used by each FWGP network. Analysis steps in the order that they are performed by each network are listed in Tables 3-5. There are inconsequential differences in the order of analysis steps used by the three networks. Each network implements its analysis steps in manners that have been generally accepted by the oceanographic and coastal engineering communities.

Table 2
Present Data Analysis Methods

Network	FCDN	NEMO	CDIP
Segmenting	Fifteen 50%-overlapping segments, 128 points each	Fifteen 50%-overlapping segments, 128 points each	50%-overlapping segments; number and length depend on record length
Mean removal method	Demean entire record	Demean each segment	Demean entire record after following mean test based on H_{mo}
Trend removal method	Linear trend removal for entire record, then 2 min. moving average demeaning of each segment	Linear for each segment	None for records < 34 min., linear trend removal for entire record for records > 34 min.
Windows	10% cosine taper for each segment	10% cosine taper for each segment	None
Correction for window use	Variance correction in frequency domain	Variance correction in frequency domain	Not needed
FFT method	IEEE (1979)	IEEE (1979)	IEEE (1979)
Frequency bands	64 bands, df = 1/128 Hz	64 bands analyzed, df = 1/128 Hz; 63 bands reported	9 bands; seven 2-sec period intervals, 4 - 18 sec, 18 - 22 sec, and 22+ sec
Upper cutoff frequency	Max. pressure correction factors = 11.18 or 31.63 depending on sensor	0.5 Hz analyzed; 0.25 Hz reported	Max. pressure correction factor = 10
Directional analysis method	Longuet-Higgins, Cartwright, and Smith (1963)	Longuet-Higgins, Cartwright, and Smith (1963)	Longuet-Higgins, Cartwright, and Smith (1963)

Initial DQA is an important first step of wave data analysis. Initial DQA consists of computerized checks for out-of-bound values, flat spots, spikes, and jumps. Each network performs these checks using techniques that they have developed, and successfully used, over the years. Some of the initial DQA procedures may pass or reject slightly more or less data than others, but none

Table 3 FCDN Data Analysis Steps
Data quality assurance (DQA)
Mean removal (entire record)
Trend removal (entire record) and use of 2-min. moving average
Segmenting
Mean removal (segments)
Windowing
FFT
Correction for window use
Cross-spectral calculations
Averaging over segments
Directional spectra calculations (including application of correction factors for subsurface measurements)
Transformation to wave direction coordinate convention
Final determination of spectra, parameters, and reporting results

Table 4 NEMO Data Analysis Steps
Data quality assurance (DQA)
Transformation to wave direction coordinate convention (PUV gages only)
Segmenting
Mean removal
Trend removal
Windowing
FFT
Correction for window use
Cross-spectral calculations
Application of pressure correction
Averaging over segments
Directional spectra calculations
Transformation to wave direction coordinate convention (slope arrays only)
Final determination of spectra, parameters, and reporting results

should pass significant numbers of poor quality data records. Additionally, each network conducts additional DQA during subsequent analysis, and a final DQA on analysis results. This document assumes that these checks are adequate and concentrates on scientific aspects of the wave data analysis itself. Development of a wave data DQA standard is left as a separate task.

Table 2 shows that the networks' analysis methods are similar from an overall perspective, but that they differ in detail. Tables 3-5 provide a similar conclusion and also show that analysis steps are, in some cases, performed in different orders. Theoretically, differing analysis step orders have little or no

Table 5 **CDIP Data Analysis Steps**
Data quality assurance (DQA)
Mean removal (entire record)
Trend removal (entire record, only for records > 34 min.)
Segmenting
FFT
Cross-spectral calculations
Averaging over segments
Directional spectra calculations (including application of correction factors for subsurface measurements)
Transformation to wave direction coordinate convention
Final determination of spectra, parameters, and reporting results

effect on final results. Differences in detail should have negligible effects on analysis results for most users of FWGP wave information.

Following parts of this section further describe the data analysis procedures and summarize their most important mathematical aspects.

Types of data and assumptions

Measured time series of directional wave data consist of digitized data for one of the following types of data sets: (a) wave pressures measured by an array of three or more pressure sensors, or (b) wave pressures measured by a single pressure sensor and wave orbital velocities in two mutually perpendicular horizontal directions. The first measured data type is called slope array data and the second measured data type is called PUV gage data. These data types are essentially equivalent and, after data analysis, produce similar results. Nondirectional wave data are measured by a single pressure sensor.

Calculation of spectra and cross-spectra from the measured time series and calculation of parameters derived from the spectra and cross-spectra provide directional and nondirectional wave information. Spectral analysis assumes that the measured time series represent stationary random processes. Ocean waves can be considered as a random process. For example, two wave records that are collected simultaneously a few wavelengths apart would have similar, but not identical, results due to statistical variability. Similarly, two wave records that are collected at slightly different times, even when wave conditions are stationary, would have similar, but not identical, results.

For the purpose of documenting FWGP wave data analysis procedures, a random process is assumed simply to be one that must be analyzed and described statistically. A stationary process is one for which actual (true) values of statistical information (e.g. significant wave height or wave spectra) are time invariant. Wave conditions are not truly stationary. However, wind wave data analysis is applied to time series using record lengths over which conditions usually change relatively little with time.

In applying statistical concepts to ocean waves, the sea surface is assumed to be represented by a superposition of small-amplitude linear waves with different amplitudes, frequencies, and directions. Statistically, individual sinusoidal wave components are assumed to have random phase angles (0 to 360 deg). To describe the frequency distribution of these wave components, wave spectra are assumed to be narrow. That is, the wave components have frequencies within a reasonably narrow range. Except for high wave conditions, the assumption of small amplitude linear wave theory is usually realistic for intermediate and deepwater waves. It is not generally true for shallow-water waves. Nevertheless, this assumption is well-known to provide suitable results for most purposes. The narrow spectrum assumption, except in some cases of swell, is almost never strictly true but also provides suitable results for most purposes. Even though these assumptions are not always valid, they provide a theoretical basis for wave data analysis procedures and help establish a framework for a consistent approach toward analysis.

Data segmenting

A measured time series can be analyzed as a single record or as a number of data segments. Data segmenting with overlapping segments decreases statistical uncertainties (i.e. confidence intervals). Data segmenting also increases spectral leakage since, for shorter record lengths in each segment, fewer Fourier frequencies are used to represent actual wave frequencies. However, for most wave data applications, spectral leakage effects are small compared to spectral confidence interval sizes.

All FWGP networks use data segmenting. Data segmenting is based on ensemble averaging of J spectral estimates from 50 percent overlapping data segments following procedures adapted from Welch (1967). Estimates from 50 percent overlapping segments generally produce better statistical properties for a given frequency resolution than do estimates from the commonly used band-averaging method (Carter, Knapp, and Nuttall 1973) without data segmenting. A measured time series, $x(n\Delta t)$ with N data points digitized at a time interval Δt is divided into J segments of length L. Each segment is defined as $x(j,n\Delta t)$, where j represents the segment number, and x indicates that values within each segment are the same as the ones that were in the equivalent part of the original time series. Following are definitions of the J segments:

$$
\begin{aligned}
&1 \quad x(1,n\Delta t) = x(n\Delta t),\ n = 0\ldots,L-1 \\
&2 \quad x(2,n\Delta t) = x(n\Delta t),\ n = \frac{L}{2},\ldots,\frac{3L}{2} - 1 \\
&J \quad x(J,n\Delta t) = x(n\Delta t),\ n = (J-1)\frac{L}{2},\ldots, \\
&\qquad (J+1)\frac{L}{2}-1
\end{aligned}
$$

where

$$
J = \frac{2\left(N-\frac{L}{2}\right)}{L}
$$

The number of segments affects confidence intervals which are discussed later. Not using segmenting is equivalent to using one segment containing all data points ($L = N$, $J = 1$).

Mean and trend removal

Means are removed for all FWGP data and means are used as an additional DQA check. As shown in Table 2, each FWGP network performs DQA based on means differently. These differences are not of major importance because the main effect of this DQA is to reject records affected by gross sensor malfunctions. Probably, such records would also be rejected by initial DQA.

Seiches, tides, and other long-period water elevation changes may produce low-frequency water elevation variations with periods longer than the length of a measured data record. These variations may appear as trends in measured data records. The FCDN and the NEMO compute trends in different manners and subtract them from the measured data (Table 2). Linear trend removal uses least-squares linear regression approaches (e.g. Bendat and Piersol (1986)). The CDIP removes trends only for records with lengths greater than 34 min. Since trends mainly introduce low-frequency noise into calculated wave spectra below regions with significant spectral density, the CDIP's nonremoval of trends for most data records should not have major effects on spectra.

Spectral leakage reduction

Spectral leakage occurs when measured wave data time series, which represent contributions from wave components with a nearly continuous distribution of frequencies, are represented by the finite number of frequencies that are used for Fourier transforms and spectral analysis. Mathematically, a data record or segment has been convolved with a boxcar window. The data begin abruptly, extend for a number of samples, and then end abruptly. The boxcar window has the advantage that all data samples have equal weight. However, the discontinuities at the beginning and the end introduce side lobes in later Fourier transforms. The side lobes allow energy to leak from the specific frequency at which a spectral estimate is being computed to higher and lower frequencies. Procedures for reducing spectral leakage are discussed in many time series analysis textbooks (e.g. Otnes and Enochson (1978), Bendat and Piersol (1980, 1986)) and lead to some disagreement in the wave measurement community.

Wave data are frequently analyzed without spectral leakage reduction. This approach is followed by the CDIP. The rationale is that leakage effects are usually small for wave parameters, such as significant wave height and peak wave period, even though spectra may differ somewhat from those that would be obtained with use of leakage-reduction techniques. Moreover, effects of leakage on spectra are generally far less than spectral confidence interval sizes. Not reducing leakage also eliminates the need for later variance corrections.

A measured wave data time series that has been Fourier-transformed provides estimates of wave component contributions at a finite number of Fourier frequencies. Leakage reduction in the frequency domain involves weighing frequency components within a moving window which is moved through all analysis frequencies. Because the variance of the data is reduced, a subsequent variance correction is made. The FWGP networks (FCDN and NEMO) that employ leakage reduction techniques use the mathematically simpler approach of applying a single window in the time domain before Fourier transforms are computed. Both networks subsequently correct for variance reduction in the frequency domain.

A cosine taper over one-tenth of each end of the data is used as a satisfactory compromise between not correcting for leakage and correcting for leakage by the most commonly used cosine bell (Hanning) approach (e.g. Childers and Durling (1975), Otnes and Enochson (1978), Bendat and Piersol (1986)). A cosine curve is applied to the first and last 10 percent of the data record or segment while the remaining 80 percent of the record is left unchanged. This procedure combines the best features of the cosine bell and boxcar windows because the sidelobes are reduced while most of the data points are given similar weight. The 10 percent cosine bell window is given by

$$W(n\Delta t) = \frac{1}{2}\left(1 - \cos\left(\frac{10\pi n}{L}\right)\right), 0 \leq n < L10$$

$$W(n\Delta t) = 1.0\ ,\ L10 \leq n \leq (L-1) - L10$$

$$W(n\Delta t) = \frac{1}{2}\left(1 - \cos\left(\frac{10\pi\left[(L-1)-n\right]}{L}\right)\right), \; (L-1) - L10 < n \le L-1$$

where L is the record or data segment length, and $L10$ is the greatest integer less than or equal to $L/10$.

The data are multiplied by the tapering function

$$x_w(j,n\Delta t) = x(j,n\Delta t) * W(n\Delta t)$$

where the subscript w indicates that the data have been windowed.

Because of the tapering at the beginning and end of data segments, the windowed data variance is less than the unwindowed data variance. To preserve the original time series variance, each data point can be multiplied by the ratio of the nonwindowed standard deviation (square root of the variance) to the windowed standard deviation. Alternatively, as performed by the FCDN and the NEMO, corrections can be made in the frequency domain by multiplying cross-spectral densities by appropriate pairs of standard deviation ratios. A standard deviation ratio is given by

$$c = \frac{\sigma}{\sigma_w}$$

where

$$\sigma = \left[\frac{1}{L-1}\left(\sum_{n=0}^{L-1} x(j,n\Delta t)^2 - \frac{\left(\sum_{n=0}^{L-1} x(j,n\Delta t)\right)^2}{L}\right)\right]^{1/2}$$

$$\sigma_w = \left[\frac{1}{L-1}\left(\sum_{n=0}^{L-1} x_w(j,n\Delta t)^2 - \frac{\left(\sum_{n=0}^{L-1} x_w(j,n\Delta t)\right)^2}{L}\right)\right]^{1/2}$$

in which x and x_w indicate the time series before and after windowing, respectively.

Fourier transforms

All FWGP networks calculate Fourier transforms with versions of a standard IEEE (1979) FFT algorithm. An FFT is a discrete Fourier transform that provides the following frequency domain representation X of a measured time series x (or x_w with use of a window).

$$X(j,m\Delta f) = \Delta t \sum_{n=0}^{L-1} x(j,n\Delta t)\, e^{-i\frac{2\pi mn}{L}}$$

where

$$m = 0,1,2,\ldots,\frac{L}{2} \qquad L \; even$$
$$m = 0,1,2,\ldots,\frac{L-1}{2} \qquad L \; odd$$

The real and imaginary parts of X are given by

$$Re[X(j,m\Delta f)] = \Delta t \sum_{n=0}^{L-1} x(j,n\Delta t) \cos\left(\frac{2\pi mn}{L}\right)$$

$$Im[X(j,m\Delta f)] = -\,\Delta t \sum_{n=0}^{L-1} x(j,n\Delta t) \sin\left(\frac{2\pi mn}{L}\right)$$

Spectral estimates are obtained at Fourier frequencies $m\Delta f$ where the interval between frequencies is given by

$$\Delta f = \frac{1}{L\Delta t}$$

FFT algorithms utilized by each network require that the number of data points be a power of two for computational efficiency so that L is even. The frequency corresponding to $m = L/2$ is the *Nyquist* frequency given by

$$f_{Nyquist} = \frac{1}{2\Delta t}$$

The Nyquist frequency is the highest resolvable frequency in a digitized time series. Spectral energy at frequencies above f_{Nyquist} incorrectly appears as spectral energy at lower frequencies.

Spectra and cross-spectra

Power spectral density (PSD) estimates for the jth segment are given by

$$S_{xx}(j,m\Delta f) = \frac{X^*(j,m\Delta f)X(j,m\Delta f)}{L\Delta t}$$

$$= \frac{|X(j,m\Delta f)|^2}{L\Delta t}$$

where X^* is the complex conjugate of X.

Cross-spectral density (CSD) estimates for the jth segment are given by

$$S_{xy}(j,m\Delta f) = \frac{X^*(j,m\Delta f)Y(j,m\Delta f)}{L\Delta t}$$

$$= C_{xy}(j,m\Delta f) - iQ_{xy}(j,m\Delta f)$$

where C_{xy} is the co-spectral density (co-spectrum), Q_{xy} is the quadrature spectral density (quadrature spectrum), and X and Y are frequency domain representations of the time series, x and y. C_{xy} and Q_{xy} can be written as

$$C_{xy}(j,m\Delta f) = \frac{Re[X]\ Re[Y] + Im[X]Im[Y]}{L\Delta t}$$

$$Q_{xy}(j,m\Delta f) = \frac{Im[X]\ Re[Y] - Re[X]Im[Y]}{L\Delta t}$$

where the arguments, $(j,m\Delta f)$, of X and Y are not shown for brevity.

Final spectral estimates are obtained by averaging the results for all segments to obtain

$$S_{xx}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} S_{xx}(j,m\Delta f)$$

$$C_{xy}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} C_{xy}(j,m\Delta f)$$

$$Q_{xy}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} Q_{xy}(j,m\Delta f)$$

With the definition of C_{xy}, the equation for S_{xx} is not needed. The co-spectra C_{xx} and C_{yy} are the same as the power spectra S_{xx} and S_{yy}. In following sections, the argument $m\Delta f$ is dropped and f is used to indicate frequency.

If data segmenting were not used, one segment ($J = 1$) would contain all of the data points in the original time series and spectral estimates would be at individual Fourier frequencies $m\Delta f$ with $L = N$ (the total number of data points in the measured time series). Spectral estimates would then be band-averaged over groups of consecutive Fourier frequencies to increase the degrees of freedom and the statistical confidence.

If frequency-dependent effects were caused by the measurement systems (e.g. sensor response, electronic filtering response), these would be corrected for at this point in the analysis. Nonzero frequency-dependent phase shifts have no effect on calculation of nondirectional wave spectra but may affect directional wave spectra. Nonunity frequency-dependent response amplitude operators affect both nondirectional and directional wave spectra. The FWGP networks have determined that there is no need to make frequency-dependent instrument-related corrections.

Frequency bandwidths are 0.0078 *Hz* for the FCDN and NEMO and 0.5 *Hz* for the CDIP. Different bandwidths do not affect significant wave heights, but have a major effect on peak wave periods. For example, near 0.10 *Hz* (10-sec period), the FCDN and NEMO periods corresponding to band center frequencies are separated by less than 1 sec compared to the 3-sec separation for CDIP center periods. CDIP bandwidths were developed originally for practical users such as boaters rather than for more technical users.

Confidence Intervals

Calculated spectra are estimates of the actual spectra. Degrees of freedom describe the number of independent variables that determine the statistical uncertainty of the estimates.

Following standard wave data analysis practice, confidence intervals are defined for PSD's, but not for co-spectra and CSD's. Confidence intervals are not provided by the FWGP networks, but may be calculated by FWGP wave information users.

There is 100 α% confidence that the actual value of a spectral estimate is within the following confidence interval

$$\left(\frac{S_{xx}(f)EDF}{\chi^2\left(EDF,\frac{(1.0-\alpha)}{2}\right)}\right),\left(\frac{S_{xx}(f)EDF}{\chi^2\left(EDF,\frac{(1.0+\alpha)}{2}\right)}\right)$$

where X^2 are percentage points of a chi-square probability distribution and *EDF* are the equivalent degrees of freedom. As noted earlier, $S_{xx} = C_{xx}$. Ninety-percent confidence intervals ($\alpha = 0.90$) are often used.

If data segmenting were not used, the equivalent degrees of freedom for a frequency band would be given by twice the number of Fourier frequencies within the band. That is

$$EDF = 2n_b$$

where n_b is the number of Fourier frequencies in the band. In general, n_b equals the bandwidth divided by the Fourier frequency interval.

The FWGP networks use data segmenting, and the *EDF* for J segments is given by

$$EDF = \frac{2J}{\left(1+\frac{0.4(J-1)}{J}\right)}$$

where J is the number of overlapping segments. The value 0.4 in the above equation is an approximation, but is adequate for segment lengths used for FWGP wave data analysis (more than 100 data points).

The FCDN and NEMO use 15 overlapping segments, which provide 22 equivalent degrees of freedom compared to the 16 degrees of freedom that would result from analysis without segmenting and with band averaging to obtain the same frequency resolution. Because of their wider frequency bandwidth, CDIP spectra have greater degrees of freedom (depending on record length) than FCDN and NEMO spectra.

Directional and nondirectional wave spectra

A directional wave spectrum provides the distribution of wave elevation variance as a function of both wave frequency f and wave direction θ. A directional wave spectrum can be written as

$$S(f,\theta) = C_{zz}(f)\ D(f,\theta)$$

where C_{zz} is the nondirectional wave spectrum (which could be determined from a wave elevation time series) and D is a directional spreading function. Integration of a directional wave spectrum over all directions (0 to 2π) provides the corresponding nondirectional spectrum given by

$$S(f) = \int_0^{2\pi} S(f,\theta) = C_{zz}(f)$$

Directional spectra are estimated using a directional Fourier series approach originally developed by Longuet-Higgins, Cartwright, and Smith (1963). Since its development, this approach has been described and used by many others (e.g. the FWGP; Earle and Bishop (1984); Steele, Lau, and Hsu (1985); Steele, Teng, and Wang (1992)). It yields directional analysis coefficients that are part of the World Meteorological Organization (WMO) WAVEOB code for reporting spectral wave information (World Meteorological Organization 1988).

The directional Fourier series approach provides the directional Fourier coefficients a_n and b_n in the following Fourier series

$$S(f,\theta) = \frac{a_o}{2} + \sum_{n=1}^{2} [a_n \cos(n\theta) + b_n \sin(n\theta)]$$

which can also be written as

$$S(f,\theta) = C_{zz}(f)\ D(f,\theta)$$

in which

$$C_{zz} = \pi\ a_o$$

and the directional spreading function is given by

$$D(f,\theta) = \frac{1}{\pi}\left(\frac{1}{2} + r_1 \cos[\theta - \theta_1] + r_2 \cos[2(\theta - \theta_2)]\right)$$

with

$$r_1 = \frac{1}{a_o}\left(a_1^2 + b_1^2\right)^{\frac{1}{2}}$$

$$r_2 = \frac{1}{a_o}\left(a_2^2 + b_2^2\right)^{\frac{1}{2}}$$

$$\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

$$\theta_2 = \frac{1}{2}\tan^{-1}\left(\frac{b_2}{a_2}\right)$$

The ambiguity of π for θ_2 is resolved by choosing the value that is closest to θ_1. The parameter θ_1 is called mean wave direction and the parameter θ_2 is called principal wave direction. These directional parameters (r_1, r_2, θ_1, and θ_2) are more commonly considered as analysis results than the parameters (a_1, b_1, a_2, and b_2) originally developed by Longuet-Higgins, Cartwright, and Smith (1963). The latter parameters are used as an intermediate calculation step.

Equations for calculation of the directional spectrum parameters follow. These equations use small-amplitude linear wave theory to correct appropriate spectral and cross-spectral values for subsurface pressure and wave orbital velocity depth attenuation factors so that calculated directional spectra have units of wave elevation variance/(Hz - radians). Directional spectra are thus spectral densities in terms of both frequency Hz and direction (radians). In the following equations, wave number k is related to frequency f by the dispersion relationship for linear waves given by

$$(2\pi f)^2 = (gk)\tanh(kd)$$

where g is acceleration due to gravity and d is mean water depth during the measurements. For a given water depth, this equation is solved for k by iterative methods.

Pressure sensor slope array:

$$a_o(f) = \frac{C_{pp}(f)}{R_p^2(f)\pi}$$

$$a_1(f) = \frac{Q_{pp_x}(f)}{R_p^2(f)k\pi}$$

$$b_1(f) = \frac{Q_{pp_y}(f)}{R_p^2(f) k \pi}$$

$$a_2(f) = \frac{(C_{p_x p_x}(f) - C_{p_y p_y}(f))}{R_p^2(f) k^2 \pi}$$

$$b_2(f) = \frac{2\ C_{p_x p_y}(f)}{R_p^2(f) k^2 \pi}$$

where p_x and p_y are pressure differences in two mutually perpendicular directions defined by the pressure sensor arrangement, and $R_p(f)$ is the wave pressure amplitude attenuation factor given by

$$R_p(f) = \frac{\cosh[k(z_d + d)]}{\cosh(kd)}$$

in which z_d is mean sensor depth beneath the sea surface (negative downward), d is mean water depth during the measurements, and k is wave number. This attenuation factor is for pressure measurements that have units of water elevation (e.g. meters). If data are in actual pressure units, the numerator of the equation for $R_p(f)$ includes the factor ρg where ρ is water density and g is acceleration due to gravity. Pressure differences can be calculated in the time domain or mathematically equivalent calculations can be made in the frequency domain during cross-spectral analysis. FWGP networks (CDIP and NEMO) that use multiple pressure gage slope arrays perform the calculations in the frequency domain.

A CDIP slope array usually consists of four pressure sensors arranged in a rectangle. Different realizations (usually five) of directional spectrum parameters from combinations of individual sensor pressure differences are calculated and averaged to yield a single set of parameters that define a directional spectrum.

PUV gage:

$$a_o(f) = \frac{C_{pp}(f)}{R_p^2(f) \pi}$$

$$a_1(f) = \frac{C_{pu}(f)}{\pi(2\pi f)\ R_u(f)R_p(f)}$$

$$b_1(f) = \frac{C_{pv}(f)}{\pi(2\pi f)\ R_p(f)R_u(f)}$$

$$a_2(f) = \frac{(C_{uu}(f)\ -\ C_{vv}(f))}{\pi(2\pi f)^2\ R_u^2(f)}$$

$$b_2(f) = \frac{2C_{uv}(f)}{\pi(2\pi f)^2\ R_u^2(f)}$$

where $R_u(f)$ is the wave horizontal velocity amplitude attenuation factor given by:

$$R_u(f) = \frac{\cosh[k(z_d + d)]}{\sinh(kd)}$$

in which the variables are the same as those for $R_p(f)$.

The direction convention for these equations is the scientific convention (e.g. Longuet-Higgins, Cartwright, and Smith 1963) which is the direction toward which waves travel measured clockwise from the x axis. Theoretically, this is the direction of the wave number vector for a particular wave component. For CDIP and FCDN data analysis, as well as NEMO's analysis of slope array data, the x axis is defined as the line between two selected pressure sensors or the direction of the u horizontal wave orbital velocity component, and transformations (i.e. rotations) to the output wave direction coordinate convention are made near the end of the analysis. The NEMO performs directional transformations for PUV gage data early in the analysis. Whether directions are transformed near the beginning or end of the analysis has no effect on final directional spectra.

This method for estimating a directional wave spectrum is described mathematically as a directional convolution of a weighing function with the actual directional spectrum. For the utilized $D(f,\theta)$ parameters, the half-power width of the weighing function is 88 deg. This width is sometimes called the directional resolution. Longuet-Higgins, Cartwright, and Smith (1963) provide a weighing of the directional Fourier coefficients to prevent unrealistic negative values of $D(f,\theta)$ for directions far from θ_1, but this approach is not used by the FWGP because it increases the half-power width to 130 deg. A directional spreading function that is determined by the described procedure is a smoothed version of the true directional spreading function. Even so, estimated directional spectra are useful. Separate directions of sea and swell can usually be identified since sea and swell often occur at different frequencies.

Higher resolution techniques, most notably Maximum Entropy Methods (MEM) and Maximum Likelihood Methods (MLM), have been developed. These techniques involve the same cross-spectral values and thus could be used. However, because they are not as widely used, have several nonstandardized versions, and may provide erroneous directional information unless they are carefully applied, they are not presently used by the FWGP except for research purposes. Benoit (1994) summarizes and compares many higher resolution techniques.

Pressure and orbital velocity signal-to-noise considerations

Low signal-to-noise ratios may occur at high frequencies due to attenuation of actual wave pressures and orbital velocities by $R_p(f)$ and $R_u(f)$. Because these attenuation factors increase with increasing frequency, unwanted amplification of high-frequency non-wave noise may occur when spectral corrections are made.

Cutoff frequency procedures for pressure gages are listed in Table 2. The maximum pressure correction factor $1/R_p(f)$ used by the FCDN is 11.18 for strain gage pressure sensors and 31.63 for Paroscientific quartz pressure sensors. Spectral and cross-spectral values at frequencies greater than or equal to the frequency where these factors are equalled or exceeded are set to zero. The CDIP has a similar pressure gage cutoff frequency except that the maximum correction factor is 10. The NEMO does not use a cutoff frequency based on the magnitude of the pressure correction factor in its analysis, but routinely deletes the highest frequency band (0.25 to 0.5 *Hz*) in its report products, since it rarely contains energy of engineering significance.

None of the FWGP networks uses a cutoff frequency procedure with wave orbital velocity data.

Different signal-to-noise considerations only affect nondirectional and directional spectra for frequencies above cutoff frequencies. Overall spectral effects should be small for the majority of wave data applications and significant wave height differences should be small except for very high frequency waves that typically have low heights.

Results and Products

Data storage

Primary results and products are information describing directional spectra from which nondirectional spectra and useful wave parameters can be calculated. Table 6 summarizes the most important data reporting products. All FWGP networks calculate directional spectra, but different information is retained for future use. Each network can use its data analysis software to reproduce results that are not retained.

Wave parameters and conventions

FWGP networks calculate several widely used wave parameters. As an example of use of wave parameters, wave climatologies may be based on statistical analysis of wave parameters on a monthly, seasonal, or annual basis. As a second example, engineering calculations of extreme wave conditions may be based on statistical analysis of wave parameters near the times of highest waves during high wave events.

Table 6
Present Data Reporting Products

Network	FCDN	NEMO	CDIP
Data storage	Time series, five Longuet-Higgins analysis coefficients as functions of frequency	Time series, cross-spectra between individual sensors, nondirectional spectra, H_{mo}, peak period, representative direction	Time series, on-line data bank of climatological statistics
Significant wave height	$H_{mo} = 4\ [m_o]^{1/2}$	$H_{mo} = 4\ [m_o]^{1/2}$	$H_{mo} = 4\ [m_o]^{1/2}$
Peak period	Corresponds to center frequency of spectral band with maximum spectral density	Corresponds to center frequency of spectral band with maximum spectral density	Corresponds to center frequency of spectral band with maximum spectral density
Representative wave direction	Mean wave direction at peak period	Mean wave direction at peak period	Spectral density weighted average of orthogonal mean wave direction components converted to direction
Wave direction convention	Direction toward which waves go relative to true north	Direction from which waves come relative to true north	Direction toward which waves go relative to true north

All FWGP networks calculate significant wave height H_{mo} from the wave elevation variance, which is also the zero moment m_o of a nondirectional wave spectrum using

$$H_{mo} = 4.0 \sqrt{m_o}$$

where m_o is computed from

$$m_o = \sum_{n=1}^{N_b} C_{zz}(f_n)\, df_n$$

in which the summation is over all frequency bands (centered at f_n) of the nondirectional spectrum and df_n is the bandwidth (constant for each FWGP network) of the nth band.

The theoretical significant wave height $H_{1/3}$ is the average height of the highest one-third waves in a wave record. An assumption for approximating significant wave height by H_{mo} is that wave spectra are narrow-banded (e.g. Longuet-Higgins (1952)). Although this assumption is not strictly valid for actual waves, considerable wave data representative of different wave conditions show that this method for determining significant wave height is suitable for nearly all purposes. Finite spectral width is the likely explanation for differences between $H_{1/3}$ and H_{mo} with H_{mo} values typically being about 5 to 10 percent greater than $H_{1/3}$ values (Longuet-Higgins 1980).

No FWGP network calculates significant wave height confidence intervals, but these can be estimated by users based on statistical approaches for estimating time series variance confidence intervals (e.g. Bendat and Piersol (1980), Donelan and Pierson (1983)). Confidence intervals depend on the total degrees of freedom (TDF), which in turn depend on spectral width. When a spectrum has been determined, the total degrees of freedom can be estimated from

$$TDF = \frac{2\left(\sum_{n=1}^{N} C_{zz}(f_n)\right)^2}{\sum_{n=1}^{N} \left(C_{zz}(f_n)\right)^2}$$

The 100 α% confidence interval for H_{mo} is given by

$$H_{m0}\left(\frac{TDF}{\chi^2\left(TDF, \frac{(1.0-\alpha)}{2}\right)}\right)^{\frac{1}{2}},\; H_{m0}\left(\frac{TDF}{\chi^2\left(TDF, \frac{(1.0+\alpha)}{2}\right)}\right)^{\frac{1}{2}}$$

where X^2 values are obtained from Chi-Square probability distribution tables. Ninety-percent intervals ($\alpha = 0.90$) are generally about -10 percent below to +15 percent above calculated H_{mo} values. These confidence intervals are smaller than those for individual spectral density values because the variance is based on information over all frequencies. The following equations provide accurate results for 90 percent ($\alpha = 0.90$) confidence intervals when *TDF* exceeds 30.

$$\chi^2(TDF,0.05) \approx TDF\left(1 - \frac{2}{9TDF} + 1.645\left(\frac{2}{9TDF}\right)^{\frac{1}{2}}\right)^3$$

$$\chi^2(TDF,0.95) \approx TDF\left(1 - \frac{2}{9TDF} - 1.645\left(\frac{2}{9TDF}\right)^{\frac{1}{2}}\right)^3$$

All FWGP networks calculate peak, or dominant, period, which is the period corresponding to the center frequency of the C_{zz} (nondirectional spectrum) spectral frequency band with maximum spectral density. That is, peak period is the reciprocal of the frequency, f_p (peak frequency), for which spectral wave energy density is a maximum. It is representative of the higher waves that occurred during the wave record. Confidence intervals for peak period are not determined and there is no generally accepted method for doing so. Peak period T_p is given by

$$T_p = \frac{1}{f_p}$$

Average and zero-crossing periods are used less often than peak period, but provide important information for some applications. These periods are not calculated by FWGP networks, but can be calculated from nondirectional spectra by the following equations:

$$T_{av} = \frac{m_o}{m_1}$$

$$T_{zero} = \left(\frac{m_o}{m_2}\right)^{\frac{1}{2}}$$

where T_{av} is average period, T_{zero} is zero-crossing period, and the spectral moments (m_0, m_1, and m_2) are given by

$$m_i = \sum_{n=1}^{n_b} f_n{}^i C_{zz}(f_n)\, df_n \qquad i = 0,1,2$$

T_{zero} is called zero-crossing period (sometimes mean period) because it closely approximates the time domain mean period which would be obtained from zero-crossing analysis of a wave elevation record. Average period and zero-crossing period each represent typical wave periods rather than periods of higher waves, which are better represented by peak period.

Among those involved in wave data collection and analysis, there are differences of opinion about the best spectral width and directional width parameters. Calculation of these parameters is left to more specialized FWGP data users. Spectral and directional width parameters can be calculated from nondirectional and directional spectra, respectively.

Many applications of FWGP data are based on a representative wave height H_{mo} wave period T_p and direction for each data record rather than the spectrum. For the FCDN and NEMO, representative wave direction is the mean wave direction θ_1 at the peak period. That is, it is the mean wave direction of the nondirectional spectrum frequency band with maximum spectral density. The CDIP performs a vector-averaged weighing of mean wave directions using spectral densities as weighing factors. Representative wave directions calculated by the FCDN and NEMO probably more closely represent directions of higher waves than those calculated by the CDIP.

As seen in Table 6, the FWGP groups use different wave direction conventions for their final results and products. Transformation to a common convention is simple, but is not presently performed.

4 Analysis Procedure Comparison and Evaluation

Analysis procedure aspects that differ between the FWGP networks and that produce negligible differences for applications of FWGP information users are: (a) normal mode record interval, (b) storm mode record interval, (c) storm mode threshold, (d) record times, and (e) analysis step order.

Some aspects that differ produce minor numerical differences for results and products. These aspects are: initial DQA, mean and trend removal, use or nonuse of leakage reduction windows, varying CDIP record lengths and sampling rates, and pressure gage cutoff frequencies. Differences caused by these aspects are not important for typical applications by FWGP information users. Scientific users of FWGP information are assumed to have sufficient expertise to appropriately use information from each network considering how data were analyzed as described in this documentation.

The wide CDIP frequency resolution may produce peak periods and spectra that differ considerably from those that would be obtained using FCDN and NEMO resolutions with the same data.

Although each network stores different information, each network can reproduce results that are not retained. The FCDN and NEMO representative wave directions probably better represent directions of higher waves, in an analogous manner to peak periods, than CDIP representative wave directions. All three networks use different wave direction conventions. Different conventions have no effect on applications, but increase possibilities for mistakes in using wave directions.

Overall, each of the FWGP networks uses analysis procedures that are generally accepted by the wave measurement and analysis community, but there are differences in procedures. Although differences in results and products should not be significant for most FWGP information users, all results and products would not be identical for the same input data. Effects would be less for users of climatological wave statistics than for users of results based on individual wave records.

5 FWGP Wave Data Analysis Standard

The standard was developed to fulfill the goals described in Chapter 2, while minimizing the expense incurred by the networks, particularly if changes would result in negligible quality improvement. The standard includes requirements that information documenting the wave data analysis be provided to the FWGP. Computer media formats for providing data, products, and related information will be developed by agreement between each network and the FWGP.

The standard does not prevent individual networks from collecting and analyzing additional data and products, provided that data and products that are furnished to the FWGP meet the standard. As examples, more data points could be collected, a shorter record interval could be used, or additional wave parameters could be calculated. However, data and products that meet the standard must be extracted from information obtained by each network.

Tables 7 through 10 describe the standard. As noted earlier, development of initial DQA procedures to be used before wave data analysis itself is performed is left as a future task.

Table 7
FWGP Data Collection Parameters

Data points per record	1,024 or 2,048, depending on site[1]
Sampling rate	1 Hz
Record interval, normal mode	1 hr
Record interval, storm mode	Not required
Storm mode threshold	Not required
Record time start	On the whole hour[2]

[1] FWGP analysis will be based on 1,024 point records where and/or when no significant energy is anticipated at swell frequencies (i. e., Great Lakes, or summer in the Gulf of Mexico) and 2,048 point records when significant swell energy is anticipated. Record length will be specified. If longer records are collected, a 2,048-sec subset will be utilized for wind-wave analysis.

[2] If existing data transmission or polling procedures preclude whole hour data without major system modifications, other record times are acceptable. Data and analysis results provided to the FWGP will be time-tagged with the actual start time of the record.

Table 8
FWGP Data Analysis Methods

Segmenting	Fifteen 50-percent overlapping segments of 128 points (1,024-sec records) Fifteen 50-percent overlapping segments of 256 points (2,048-sec records)
Mean removal	Entire record and each segment
Trend removal	Linear for each segment
Windowing	10-percent cosine taper for each segment
Correction for windowing	Variance correction in frequency domain
FFT	IEEE (1979)
Frequency bands	64 bands, bandwidth (df) = 1/128 Hz (1,024-sec records) 128 bands, bandwidth (df) = 1/256 Hz (2,048-sec records)
Pressure gage cutoff frequency	Sensor dependent[1]
Directional analysis	Longuet-Higgins, Cartwright, and Smith (1963)

[1] Each network will establish and document a minimum pressure value or a maximum subsurface pressure to elevation correction value that applies to their instrumentation and will use these values during data analysis. Values for all instrumentation types will be provided to the FWGP. Analysis results for each wave record will indicate whether a pressure gage cutoff frequency was applied and, if so, will include the frequency.

Table 9 FWGP Data Analysis Steps
Data quality assurance (DQA)
Mean removal (entire record)
Segmenting
Mean removal
Trend removal
Windowing
FFT
Correction for window use
Cross-spectral calculations
Averaging over segments
Directional spectra calculations (including application of correction factors for subsurface measurements)
Transformation to wave direction coordinate convention
Final determination of spectra, parameters, and reporting results
Use of five directional analysis coefficients, H_{mo}, T_p, and representative wave direction with the noted direction convention is consistent with requirements of the World Meteorological Organization (WMO) WAVEOB code for reporting spectral wave information (World Meteorological Organization, 1988). Zero-crossing wave period T_{zero} is optional for the WAVEOB code, but it is useful in addition to peak period, because it represents an average or typical wave period for an analyzed wave record.

Table 10 FWGP Data Reporting Products	
Date/Time of Record	Julian day/UCT time at beginning of record
Spectral Products	Five directional analysis coefficients as functions of frequency
Significant wave height	$H_{mo} = 4\ [m_o]^{1/2}$ in m
Peak period	T_p corresponds to center frequency of spectral band with maximum spectral density
Zero-crossing period	$T_{zero} = [m_o / m_2]^{1/2}$
Representative wave direction	Mean wave direction at peak period
Wave direction convention	Direction from which waves come, in degrees relative to true north
Mean depth to seabed	Mean pressure for record, converted to meters and corrected for sensor height, in meters

6 Summary and Acknowledgements

Summary

Wave data analysis procedures used by FWGP networks are documented, compared, and evaluated and the FWGP wave data analysis standard is defined. Wave data analysis assumptions are noted and appropriate mathematical equations are provided as a guide for present and future FWGP networks. Procedures that can be applied by FWGP wave information users to obtain additional useful information (e.g. confidence intervals) from FWGP results and products are described.

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CDIP (SIO)

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NEMO (CERC)

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Appendix A Definitions

Confidence intervals: Spectra and wave parameters (e.g. significant wave height) have statistical uncertainties due to the random nature of waves. For a calculated value of a given spectral density or wave parameter, confidence intervals are values less than and greater than the calculated value within which there is a specific estimated probability for the actual value to occur.

Cross-spectral density (CSD): Cross-spectral density represents the variance of the in-phase components of two time series (co-spectrum) and the variance of the out-of-phase components of two time series (quadrature spectrum). Phase information contained in cross-spectral density estimates helps determine wave directional information.

Deepwater waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be deepwater waves when $d/L > 1/2$.

Directional spreading function: At a given frequency, the directional spreading function provides the distribution of wave elevation variance with direction.

Directional wave spectrum: A directional wave spectrum describes the distribution of wave elevation variance as a function of both wave frequency and wave direction. The distribution is for wave variance even though spectra are often called energy spectra. For a single sinusoidal wave, the variance is 1/2 multiplied by wave amplitude squared and is proportional to wave height squared. Units are those of energy density = amplitude^2 per unit frequency per unit direction. Units are usually $m^2/(Hz$-degree) but may be $m^2/(Hz$-radian). Integration of a directional wave spectrum over all directions provides the corresponding nondirectional wave spectrum.

Intermediate (transitional) water depth waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be intermediate, or transitional, waves when $1/25 < d/L < 1/2$.

Leakage: Leakage occurs during spectral analysis because a finite number of analysis frequencies are used even though the measured time series has contributions over a nearly continuous range of frequencies. Because a superposition of wave components at analysis frequencies is used to represent a time series with actual wave components at other frequencies, variance may appear at incorrect frequencies in a wave spectrum.

Mean wave direction: Mean wave direction is the average wave direction as a function of frequency. It is mathematically described by $\theta_1(f)$.

Measured time series: A sequence of digitized measured values of a wave parameter is called a measured time series. For computational efficiency reasons, the number of data points is equal to 2 raised to an integer power (e.g. 1,024, 2,048, or 4,096 data points). A measured time series is also called a wave record.

Nondirectional wave spectrum: A nondirectional wave spectrum provides the distribution of wave elevation variance as a function of wave frequency only. The distribution is for wave variance even though spectra are often called energy spectra. It can be calculated by integrating a directional wave spectrum over all directions for each frequency. Units are those of spectral density which are amplitude$^2/Hz$ (e.g. m^2/Hz) (see also directional wave spectrum).

Nyquist frequency: The Nyquist frequency is the highest resolvable frequency in a digitized measured time series. Spectral energy at frequencies above the Nyquist frequency appears as spectral energy at lower frequencies. The Nyquist frequency is also called the folding frequency.

Peak (dominant) wave period: Peak, or dominant, wave period is the wave period corresponding to the center frequency of the frequency band with the maximum nondirectional spectral density. Peak wave period is also called the period of maximum wave energy.

Power spectral density (PSD): See spectral density.

Principal wave direction: Principal wave direction is similar to mean wave direction, but it is calculated from other directional Fourier series coefficients. It is mathematically described by $\alpha_2(f)$.

Random process: Measured wave characteristics (e.g. wave elevation, pressure, orbital velocities) represent a random process in which their values vary in a nondeterministic manner over a continuous period of time. Thus, descriptions of wave characteristics must involve statistics.

Representative wave direction: Representative wave direction for a measured time series is the mean wave direction for the frequency band with the maximum nondirectional spectral density (FWGP standard, FCDN, and NEMO). It is also a spectral density weighted vector-average of all mean wave directions (CDIP). The first definition is more commonly used in wave data analysis.

Sea: Sea consists of waves which are observed or measured within the region where the waves are generated by local winds.

Shallow-water waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be shallow-water waves when $d/L < 1/25$.

Significant wave height, H_{mo}: Significant wave height H_{mo} can be estimated from the variance of a wave elevation record assuming that the nondirectional spectrum is narrow. The variance can be calculated directly from the record or by integration of the spectrum as a function of frequency. Using the latter approach, H_{mo} is given by $4(m_o)^{1/2}$ where m_o is the zero moment of nondirectional spectrum. During analysis, pressure spectra are converted to equivalent sea surface (elevation) spectra so that these calculations can be made. Due to the narrow spectrum assumption, H_{mo} is usually slightly larger than significant wave height $H_{1/3}$ calculated by zero-crossing analysis.

Significant wave height, $H_{1/3}$: Significant wave height $H_{1/3}$ is the average crest-to-trough height of the highest one-third waves in a wave record. Historically, $H_{1/3}$ approximately corresponds to wave heights that are visually observed. If a wave record is processed by zero-crossing analysis, $H_{1/3}$ is calculated by actual averaging of the highest one-third heights. H_{mo} is generally used in place of $H_{1/3}$ since spectral analysis is more often performed than zero-crossing analysis and numerical wave models provide wave spectra rather than wave records.

Small-amplitude linear wave theory: Several aspects of wave data analysis assume that waves satisfy small amplitude linear wave theory. This theory applies when ak, where a is wave amplitude and k is wave number (2π/wavelength), is small. Wave amplitudes must also be small compared to water depth. Except for waves in very shallow water and very large waves in deep water, small-amplitude linear wave theory is suitable for most practical applications.

Spectral density: For nondirectional wave spectra, spectral density is the wave elevation variance per unit frequency interval. For directional wave spectra, spectral density is the wave elevation variance per unit frequency interval and per unit direction interval. Values of a directional or nondirectional wave spectrum have units of spectral density. Integration of spectral density values over all frequencies (nondirectional spectrum) or over all frequencies and directions (directional spectrum) provides the total variance of wave elevation (see also directional wave spectrum and nondirectional wave spectrum).

Spectral Moments: Spectral moments are used for calculations of several wave parameters from wave spectra. The rth moment of a wave elevation spectrum is given by

$$m_r = \sum_{n=1}^{n_b} (f_n)^r C_{zz}(f_n) \Delta f_n$$

where Δf_n is the spectrum frequency band width, f_n is frequency, C_{zz} is nondirectional spectral density, and n_b is the number of frequency bands in the spectrum.

Stationary: Waves are considered stationary if actual (true) statistical results describing the waves (e.g. probability distributions, spectra, mean values, etc.) do not change over the time period during which a measured time series is collected. From a statistical viewpoint, actual waves are usually not truly stationary, even though they are assumed to be so for analysis.

Swell: Swell consists of waves that have travelled out of the region where they were generated by the wind. Swell tends to have longer periods, more narrow spectra, and a more narrow spread of wave directions than sea.

Wave amplitude: Wave amplitude is the magnitude of the elevation of a wave from mean water level to a wave crest or trough. For a hypothetical sine wave, the crest and trough elevations are equal and wave amplitude is one-half of the crest-to-trough wave height. For actual waves, especially high waves and waves in shallow water, crests are further above mean water level than troughs are below mean water level, so crest-to-trough wave height is a more useful parameter.

Wave crest: A wave crest is the highest part of a wave. A wave trough occurs between each wave crest.

Wave energy: For a single wave, wave energy per unit area of the sea surface is $(1/2)\rho g a^2$ where ρ is water density, g is acceleration due to gravity, and a is wave amplitude. The constant factor, ρg, is usually not considered so that energy or wave variance is considered as $(1/2)\ a^2$. Since wave components have different frequencies and directions, wave energy can be determined as a function of frequency and direction.

Wave frequency: Wave frequency is 1/(wave period). Wave period is measured in units of seconds, and wave frequency is measured in units of hertz, which is the same as cycles/second. Radian wave frequency (circular frequency) is $2\pi f$ where f is frequency in hertz.

Wave height: Wave height is the vertical distance between a wave crest and a wave trough. Wave height is approximately twice the wave amplitude. However, for large waves and waves in shallow water, wave crests may be considerably further above mean sea level during the time of the measurements than crests are below mean sea level. When using a spectral analysis approach, estimates of wave height are calculated from spectra using well-established wave statistical theory instead of being calculated directly from a measured wave elevation time series.

Wavelength: Wavelength is the distance between corresponding points on a wave profile, such as the distance between successive crests or troughs.

Wave number: Wave number is defined as 2π/wavelength.

Wave period: Wave period is the time between corresponding points on a wave profile passing a measurement location. It can be the time between crests, between troughs, or between zero-crossings (mean sea level crossings). The distribution of wave variance as a function of wave frequency (1 / period) can be determined from spectral analysis so that tracking of individual wave periods from wave profiles is not necessary when wave spectra are calculated.

Wave trough: A wave trough is the lowest part of a wave. A wave crest occurs between each wave trough.

Zero-crossing wave period: Zero-crossing wave period is the average of the wave periods that occur in a wave height time series record where a wave period is defined as the time interval between consecutive crossings in the same direction of mean sea level during a wave measurement time period. It is also statistically the same as dividing the measurement time period by the number of waves. An estimate of zero-crossing wave period can be computed from a nondirectional spectrum. This estimate is statistically the same as averaging all wave periods that occurred in the wave record.

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Marshal D. Earle, David McGehee, Michael Tubman

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Neptune Sciences, Inc., 11341 Sunset Hills Road, Reston, VA 22090
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This version of the Field Wave Gaging Program (FWGP) Wave Data Analysis Standard focuses on procedures for analyzing directional wave data from pressure slope arrays and pressure/biaxial current meter gages. Nondirectional wave data analysis is inherently included as a simplification (a subset) of directional wave data analysis. Likewise, analysis of data from wave staffs is a subset of these procedures, obtained by eliminating the pressure response correction. The data collection and analysis procedures described are only applicable to wind-generated surface gravity waves of engineering significance. This document covers analysis procedures for measured time series (wave records) that are assumed to contain no significant number of errors or gaps.

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